41

ON TWO DIRECTIONAL FOCUSSING MAGNETIC ANALYSERS

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ABSTRACT. Design parameters for a symmetrical two directional non-uniform magnetic analyser are considered. Suggestions are made for achieving second order radial focussing effect by properly shaping the pole edges of the first order focussing conical magnets having an angle $< \sqrt{2}\pi$. The advantage of such a 180 magnet for mass-spectrometry is discussed in consideration with the resolving power and solid angle.

Since the introduction of a non-uniform magnetic field for decreasing the radial and axial oscillations of electrons in a betatron, magnetic analysers using inhomogeneous magnetic fields have been important for improved focussing of charged particles in β-ray spectrometers, nuclear spectrometers and mass- spectrometers. The field shape of a betatron magnet, derived by Kerst and Serber (1941) is

$$H = H_0 \left(\frac{a}{r}\right)^n \quad \ldots \ (1)$$

where H_0 is the field at the equilibrium orbit of the charged particles of radius a, and the field index $n = -\frac{r}{H}\frac{\partial H}{\partial r}$ of the axially varying field H.

Expanding equation (1) according to Shull and Dennison (1947) we get the series, representing the vertical component of such a field,

$$H_z = H_0(1-\alpha\delta+\beta\delta^2 \ldots) \quad \ldots \ (2)$$

where $\alpha = n$, $\beta = \frac{n^2+n}{2}$, $\delta = \frac{r-a}{a}$, where $r = a+dr$. Since curl $H = 0$, the radial component of the field will be

$$Hr = 2H_0\left[-\frac{\alpha}{a}+\frac{2\beta(r-a)}{a^2}\right] \quad \ldots \ (3)$$

For two directional focussing of charged particles with such a non-uniform field, n should be greater than zero and less than unity. The angular frequency of the radial and axial oscillation in such a field can be shown to be $\omega_r = (1-n)^{\frac{1}{2}}\omega_0$ and $\omega_a = n^{\frac{1}{2}}\omega_0$ respectively, where ω_0 is the orbital frequency of the accelerated charged particle. Putting $n = \frac{1}{2}$, we have $\omega_r = \omega_a$ and the image is focussed at an angle $\sqrt{2}\pi$.

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Siegbahn and Svartholm (1946) have constructed a β-ray spectrometer with such a non-uniform magnet and now these magnetic analysers are widely used in β-ray spectrometry. From the expression (2) it is evident that a perfect magnet pole face of this type, satisfying higher order focussing, is difficult to achieve. So the nearest approaches have been attempted by different authors (Vers ter, 1950 and Siegbahn *et at* (1946).

Taking the first term of the expression (2), first order two directional focussing of charged particles can, conveniently, be achieved. In this case, the pole face has a conical shape, the cone angle η being equal to $\tan^{-1} \frac{nb}{a}$, where $2b$ is the gap width of the magnet at the equilibrium orbit of radius a, field index $n = \frac{1}{2}$. For achieving higher resolving power, elimination of second order aberration is to be considered. Shull and Dennison (1947) have shown that for $\beta = 3/8$ or $1/8$ the second order aberration in axial or radial focussing respectively, can be eliminated. As there is no single value of β for second order focussing in both the directions, Rosenblum (1950) suggests the value of β equal to $1/4$ for an average better focussing in both the directions.

Focussing of charged particles at an angle $\sqrt{2}\pi$ with this non-uniform field is equivalent to a single directional focussing homogeneous 180° magnetic analyser after Dempster (1918) in the respect that the source and detector are to be placed within the magnetic field. Though the weight of magnetic material is minimised by using a ring magnet in the case of non-uniform magnetic analysers, yet the convenience of placing the source and detector out of the magnetic field can not be denied. Judd (1950) and Rosenblum (1950) have, independently, worked out the feasibility of focussing of the charged particles with such a non-uniform magnetic field at a smaller angle than $\sqrt{2}\pi$. Synder and others (1950) have used a 180° non-uniform first order focussing magnetic analyser for nuclear spectrometry. They have used such a magnetic analyser with source and detector placed at unequal distances from the pole boundary. So far the design of a symmetrical magnetic analyser of this type, having a focussing angle less than $\sqrt{2}\pi$, is not reported. But for mass-spectrometry, where focussing at a constant radius is important, a symmetrical non-uniform analyser is very convenient.

For $n = \frac{1}{2}$, Rosenblum has worked out the following expression for a first order focussing magnetic analyser.

$$l'' = \frac{l'/n^{\frac{1}{2}}(\cot\, n^{\frac{1}{2}}\phi)+\frac{1}{n}}{l'-\left(\frac{1}{n^{\frac{1}{2}}}\right)\cot\, n^{\frac{1}{2}}\phi} \qquad \ldots \quad (4)$$

where l' and l'' are the distances of the source and the detector, respectively, from pole edges in units of radius of curvature a of the equilibrium orbit, ϕ is the the focussing angle. We may derive an expression for $l = l' = l''$ representing the distance of such a magnetic anzlyser of symmetrical type from (4) and rejecting the negative term.

$$l = \sqrt{2}\ \frac{\cos\phi/\sqrt{2}+1}{\sin\phi/\sqrt{2}} \quad \text{for } n = \tfrac{1}{2} \qquad \ldots \quad (5)$$

Putting $\phi = \sqrt{2}\pi$ in exp. (5) l becomes zero which is the case deduced by Shull and Dennison (1947) and Svartholm (1946) independently.

In the same expression

when $\qquad \phi = \pi, \quad l = \cdot 7 \qquad \ldots \quad (6)$

and for $\qquad \phi = \pi/2, \quad l = 2.3 \qquad \ldots \quad (7)$

Here the values of l are in units of radius of curvature a. The correctness of these values can independently be checked from the expression, derived from Judd's (1950)

$$l_2 = -\sqrt{2}\,a\ \tan\ [\phi/\sqrt{2}+\tan^{-1} l_1/\sqrt{2}a]$$

where l_1 and l_2 are distances in centimeters of the source and detector respectively from the magnet.

For focussing of charged particles with a first order focussing effect non-uniform magnetic fields having an angle smaller than $\sqrt{2}\pi$ can easily be achieved by shaping the pole faces conically as described before. For second order focussing, instead of shaping the pole face for a suitable value of β, we may attempt to eliminate the radial second order aberration by shaping the pole edges as given by Hintenberger (1948) for the single directional focussing magnetic analyser

According to him, with a magnetic analyser satisfying the first order focussing condition, the condition for second order focussing is

$$a\left(\frac{\gamma_1}{R_1}+\frac{\gamma_2}{R_2}\right) = c_1+c_2 \qquad \ldots \quad (9)$$

where a is the radius of curvature of the orbit of the particle, and the constants

$$\gamma_1 = \frac{1}{\cos^3\epsilon_1\left\{\left[1+\left(\frac{a}{l_1}+\tan\epsilon_1\right)^2\right]^3\right\}^{\frac{1}{2}}}$$

$$\gamma_2 = \frac{1}{\cos^3\epsilon_2\left\{\left[1+\left(\frac{a}{l_2}+\tan\epsilon_2\right)^2\right]^3\right\}^{\frac{1}{2}}}$$

$$c_1 = \frac{a^2}{l_1^2}\ \frac{a/l_1+3\tan\epsilon_1}{\left\{1+\left(\frac{a}{l_1}+\tan\epsilon_1\right)^2\right]^3\right\}^{\frac{1}{2}}}$$

$$c_2 = \frac{a^2}{l_2^2}\ \frac{(a/l_2+3\tan\epsilon_2)}{\left\{\left[1+\left(\frac{a}{l_2}+\tan\epsilon_2\right)^2\right]^3\right\}^{\frac{1}{2}}}$$

Here R_1, R_2 are the radii of curvature of the entrance and exit boundary of the field, ϵ_1, ϵ_2 are the angles made by the central beam of the particles with the normal at the entrance and exit of the pole boundary, respectively, a is the radius of curvature of the path of the particle.

From the above expression (9) it is possible to construct two types of sector-shaped magnetic spectrometers as follows:

1. *Normal cirle type* : In this type $\epsilon_1 = \epsilon_2 = 0$ and pole edges are circular having radii of curvature R_1 and R_2 at entrance and exit, respectively.

2. *Inflection type* : Kerwin (1949) derived an expression for the pole edge for perfect focussing, which is a curve, having a point of inflection. Linear approximation of such curves, drawing a straight line through this point of inflection, can be done so as to achieve a second order focussing effect. This may be compared with the expression (9) when $R_1 = R_2 = \infty$ and $\epsilon_1, \epsilon_2 \neq 0$.

In the gap of a homogeneous magnet, $H_x = H_y = 0$, H_z component in z-direction is responsible for the radial focussing of charged particles But at the entrance and exit of the magnet the fringing field has a finite H_y component, which may axially defocus the charged particles in case of the inflection type spectrometers. So the normal circle type, where $\epsilon_1 = \epsilon_2 = 0$ and thus $H_y = 0$ in the fringing field region, is better than the inflection type.

In the two directional focussing magnetic analysers, radial focussing is responsible for the resolving power and the axial focussing for intensity. For $n = \frac{1}{2}$, in a non-uniform magnetic analyser having a conical pole face first order radial focusing condition is satisfied and Hintenberger's formula for second order radial focussing can conveniently be applied. So by shaping the pole edges, increased resolving power can be attained for the same solid angle. From (9) we have $R = l^3$ where $\epsilon_1 = \epsilon_2 = 0$ and $l' = l'' = l$, $R_1 = R_2 = R$ for symmetrical cases, R and l being in units of radius of curvature a.

Putting the values of l for $\phi = \pi$ and $\pi/2$ from (6) and (7) we have
$R = .343$ and 12.17 for the said focussing angles respectively.

In the case of a non-uniform magnet having an angle of focussing $\sqrt{2}\pi$ the conjugate foci lie in the field. So second order correction here is only possible by choosing a suitable value of β in eqn. (2). But for all the magnets having an angle $< \sqrt{2}\pi$ second order focussing can be achieved by shaping the pole edges as described. Radii of curvature of the pole edges for such magnets are derived for the two specific cases, i.e. $\phi = \pi$ and $\pi/2$. It is evident that intensity decreases for greater l. Moreover for increased l, the radii of cuvature of pole edges are so large, that such shaping requires a more precision craftmanship.

Though for all angles $< \sqrt{2}\pi$, the pole edges of this type of non-uniform magnetic analysers can be properly shaped for second order focussing effect, yet $\phi = \pi$ is preferred for high intensity mass spectrometry because of the fact that the pole faces can be machined simultaneously and the source and the detector can conveniently be placed outside the magnetic field. For $\phi = \sqrt{2}\pi$, the first order focussing magnetic analyser has a maximum solid angle for a given resolving power. In the case of a magnetic analyser having $\phi = \pi$, the decreased intensity can be compensated by utilising the above mentioned second order focussing device for the same resolving power or when a better resolving power is essential at the cost of solid angle, such 180° magnetic analysers, having shaped pole edges, are superior to those having $\phi = \sqrt{2}\pi$.

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